Thus, it can be concluded that with a confidence probability of P = 0.95, the desired value of deflection angle from the gas-dynamic prism for a launch coordinate x_0 = 2.4 mm deviates from the measured average value α = 2.95° by an amount no greater than 0.078° = 4.7'. The relative measurement error was 2.5%.

CONCLUSIONS

The theoretical and experimental studies conducted on the gas-dynamic prism show that it can be used more effectively than traditional aerooptical elements for deflecting a light beam at large angles. However, the degradation in the beam divergence of the light in the direction transverse to the axis of the nozzle requires one to perform some compensating measures to correct it. In the latter case, in all probability, the most suitable approach is to use a gas-dynamic system for beam deflection in combination with well-known thermal concentration and diffusion compensation methods in the transverse direction; this allows for the fact that the perturbation of the angles introduced by the prism in this direction are small.

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EXISTENCE OF STATIONARY WAVES OF RADIATION COOLING

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The solution of the system of equations of gas dynamics and radiation transfer is analyzed and it is shown that a Zel'dovich–Raizer stationary wave of radiation cooling does not exist in a hot gas.

The radiation cooling of a hot volume of air was studied in [1, 2]. It was shown that because of the extremely sharp temperature dependence of the optical properties of air such cooling must occur in the form of a temperature step propagating in the hot air – a so-called wave of cooling (WC) [3]. Cooling by radiation in this manner drops the air temperature from 10^5-10^6 K and higher to ~10^4 K within a short time. Assuming that the velocity of the wave of cooling is low compared with the velocity of sound, the authors neglected, owing to its smallness, the pressure jump on the front of the WC and the air motion arising, and instead of considering the complete system of equations of gas dynamics and radiation transfer, which describe this process, they limited their analysis to the energy equation and the radiation transfer equation. After integrating these equations,

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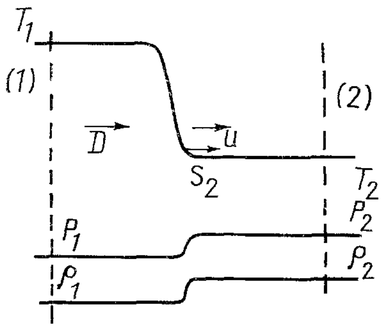

Fig. 1. Schematic diagram of the profiles of the temperature, pressure, and density in the wave of cooling.

under the assumption that the WC is quasi-stationary, Zel'dovich et al. obtained in [1, 2] an analytic expression for the temperature profile on the front of the WC and they found the radiation flux from the wavefront and the propagation velocity of the flux.

It is obvious, however, that since the density of the cold gas is higher than that of the hot gas, such cooling gives rise to motion of matter from the periphery toward the center, i.e., a gas-dynamic flow of air will arise. For this reason, it is more correct to study the complete system of equations of gas dynamics and radiation transfer.

The aim of this work is to solve the complete system of equations for this problem under the assumption that the wave of cooling is stationary. From analysis of the final states, which are the solution of this complete system of equations, it can then be concluded whether or not the assumptions made in [1, 2] are correct and also the character of the stationary wave of cooling can be judged.

We study a one-dimensional stationary WC, propagating in hot air with velocity D. We shall employ the result of [1] that the radiation flux S_2 flowing out of the hot region is generated in the trough of the wave and is equal to approximately σT_2^4, where $T_2 \approx 10^4$ K is the transparency temperature of air. We shall also assume that absorption in the region not too far behind the front of the WC can be neglected. Under these conditions, the standard integrals of the equations of gas dynamics in the system of coordinates moving with the velocity of the front of the WC exist (Fig. 1):

$$\rho_1 D = \rho_2 u = C_1, \tag{1}$$

$$\rho_1 D^2 + P_1 = \rho_2 u^2 + P_2 = C_1 C_2, \tag{2}$$

$$W_1 + \frac{D^2}{2} = W_2 + \frac{u^2}{2} + \frac{S_2}{\rho_1 D}, \tag{3}$$

The indices 1 and 2 denote quantities in some sections (1) and (2) (see Fig. 1) ahead of and behind the WC front, respectively.

We assume that air is an ideal gas with heat-capacity ratio $\gamma = c_p/c_V$. In the range of air temperatures studied $\gamma \approx 1.15$-1.4 [3]. To the system of equations (1)-(3) we add the equation of state of a perfect gas

$$P = A\rho T, \tag{4}$$

where $A = R/\mu_1$ is the gas constant, calculated per 1 g, as well as the thermodynamic relation

$$W = c_p T = \frac{\gamma}{\gamma - 1}\frac{P}{\rho}. \tag{5}$$

It follows from Eqs. (1) and (2) that in the process of compression in the absence of viscosity the change in the state of an air particle as it passes through the front of the WC should occur along the straight line $P = P_1 + \rho_1 D^2(1 - X)$, where $X = \rho_1/\rho = V/V_1$ in the (P, V) plane.

We shall find the relation between the initial and final states of the gas in the variables (P, V), for which with the help of Eqs. (1), (2), (4), and (5) we eliminate from Eq. (3) W_1, W_2, u, and D. We also take into account the fact that $S_2 \approx \sigma T_2^4$. As a result we obtain an expression analogous to the equation of a shock adiabat:

$$P_2\left(\frac{\gamma+1}{\gamma-1}V_2-V_1\right)-P_1\left(\frac{\gamma+1}{\gamma-1}V_1-V_2\right)+\frac{2\sigma}{A^4}P_2^4V_2^4\sqrt{\frac{V_1-V_2}{P_2-P_1}}=0. \tag{6}$$

In addition to Eq. (6), the relation

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2} \tag{7}$$

should hold at the point of the final state in the (P, V) diagram.

It is convenient to write the expressions (6) and (7) in the variables $Y = P_2/P_1$ and $X = V_2/V_1$. This gives

$$Y\left(\frac{\gamma+1}{\gamma-1}X-1\right)-\frac{\gamma+1}{\gamma-1}+X+\frac{2\sigma}{A^4}P_1^{2,5}V_1^{3,5}\sqrt{\frac{1-X}{Y-1}}X^4Y^4=0, \tag{8}$$

$$XY=\frac{T_2}{T_1}=\frac{10^4}{T_1}. \tag{9}$$

Thus the final states of air after the passage of the WC are represented in the (X, Y) plane by the point of intersection of the curves (8) and (9) with the initial parameters P_1, V_1. It is important to note that since the final temperature T_2 is determined by the optical properties of air and is known beforehand, the system of equations (8) and (9) with the initial values P_1, V_1 has a unique solution, i.e., a point in the (X, Y) diagram. For greater clarity we shall represent the set of points of final states with different initial conditions by a curve with the parameter P_1, for which we substitute Eq. (9) into Eq. (8) and introduce the dimensionless pressure $P'_1 = P_1/10^6$. We also set $\gamma = 1.4$. The final equation that expresses the set of points of final states in the (X, Y) plane as a function of the initial pressure P'_1 has the form

$$\frac{6{,}7}{P_1'}=(6-X-6XY+Y)\sqrt{\frac{Y-1}{XY(1-X)}}. \tag{10}$$

We find the other initial parameter V_1 from Eq. (9) with the values of (X, Y) of the point of the final state.

Figure 2 shows curves of the final states (10) with the initial pressures P_1 = 1, 2, and 10 atm. Since we are investigating a wave of cooling, the condition $T_2 < T_1$ or $XY < 1$ in the (X, Y) variables should hold for the final states. These points lie beneath the curve 5 (Fig. 2). From Fig. 1 one can see that final states where $P_1 = P_2$ (Y = 1) do not exist. As is well known, a finite jump in the pressure on the wavefront in a nonviscous gas generates a wave which propagates through the undisturbed gas with supersonic velocity $D > c_1$. This means that the points lying below the straight line 6 (Fig. 2), satisfying the condition $D = c_1$, cannot be realized. Thus the points of the final states, satisfying the conditions listed above, can be located only in the region below the curve 5 and above the straight line 6.

We shall now determine whether or not a solution of the radiation transfer equation, which admits such a process, exists. For this, it is convenient to employ the phase-plane method [4]. For a stationary one-dimensional process the energy equation can be written in the form [5]

$$\rho u\frac{d}{dx}\left(W+\frac{u^2}{2}\right)=\frac{dS}{dx}, \tag{11}$$

$$\frac{dS}{dx}=4\pi\int_0^\infty \varkappa_\nu\left(\frac{1}{2}\int_{-1}^{1}I_\nu d\mu-B_\nu\right)d\nu,$$

$$B_\nu=\frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1}. \tag{12}$$

As a simplification, instead of the exact radiation transfer equation we shall study the diffusion approximation of this equation, and we shall take into account the spectral

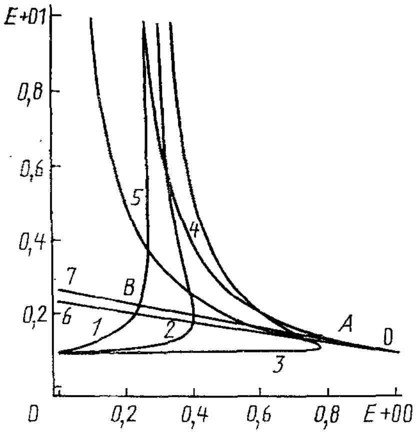

Fig. 2

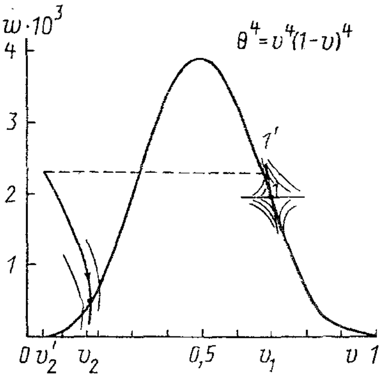

Fig. 3

Fig. 2. Curves of the final states with P_1 = 1, 2, and 10 atm (1, 2, and 3, respectively), the Hugoniot adiabat for a perfect gas with γ = 1.4 (4), a plot of the function XY = 1 (5), a straight line whose slope corresponds to the velocity of sound c_1 in a hot gas (6), and one of the possible straight lines connecting the initial zero and the final state B of a particle of air (7).

Fig. 3. Picture of the integral curves for the process depicted by the straight line 7 in Fig. 2; M_1 = 1.2.

composition of the radiation by introducing an appropriately spectrum-averaged mean-free path length ℓ of the radiation [1]:

$$S = -\frac{lc}{3}\frac{dU}{dx}. \tag{13}$$

As the independent variable we introduce the optical thickness τ: $d\tau = \kappa dx$. In the case of an ideal gas it is convenient to introduce the following normalization: $v = u/C_2$ and $\omega = (\pi A^4/2\sigma C_2{}^8)I(x, \mu)$. In these variables, after integrating over the spectrum. Eqs. (11) and (13) assume the following form [5]:

$$b\frac{d}{d\tau}\left(\frac{\gamma}{\gamma+1}v - \frac{v^2}{2}\right) = w - \theta^4, \tag{14}$$

$$\frac{1}{3}\frac{d^2w}{d\tau^2} = w - \theta^4, \tag{15}$$

where

$$b = \frac{\rho_1 A^4(\gamma+1)}{4\sigma D^5(\gamma-1)}\left(1 + \frac{1}{\gamma M_1^2}\right)^{-6};\ \theta = v(1-v).$$

Replacing in Eq. (15) the right-hand side by the left-hand side of Eq. (14) and integrating once over τ, we obtain the first integral:

$$\frac{1}{3}\frac{dw}{d\tau} + C = b\left(\frac{\gamma v}{\gamma+1} - \frac{v^2}{2}\right). \tag{16}$$

The integration constant C can be found from the boundary conditions ahead of the front of the WC

$$\tau = -\infty;\ S_1 = 0;\ U_1 = U_{1e} = \frac{4\sigma T_1^4}{c},$$

or behind the front of the WC

$$\tau \approx 0;\ S_2 \approx \sigma T_2^4;\ U_2 = U_{2e} = \frac{4\sigma T_2^4}{c}.$$

We note that for the point 2 behind the front of the WC we took the point where $S_2 \approx \sigma T_2^4$, $U_2 = U_{2e}$. In [2] it is shown that such a point exists and it is convenient to take this point as the bottom edge of the WC.

Converting from differentiation with respect to τ to differentiation with respect to v, we obtain the equation

$$\frac{dw}{dv} = 3b\left[b\left(\frac{\gamma v}{\gamma+1} - \frac{v^2}{2}\right) - C\right]\frac{v_s - v}{w - \theta^4}, \tag{17}$$

where $v_s = \gamma/(\gamma + 1)$.

Taking into account the boundary conditions we find the following expressions for the integration constant C:

$$C = b\left(\frac{\gamma v_1}{\gamma+1} - \frac{v_1^2}{2}\right) = b\left(\frac{\gamma v_2}{\gamma+1} - \frac{v_2^2}{2}\right) + \frac{S_2 A^4}{4\sigma C_2^8}. \tag{18}$$

Substituting the expressions for C at the points 1 and 2 in Eq. (17), we obtain the final differential equations of the problem in the phase plane (v, w) in the vicinity of the initial point

$$\frac{dw}{dv} = \frac{3b^2}{2}\,\frac{(v_s - v)(v_1 - v)}{w - \theta^4}\left(v_1 + v - \frac{2\gamma}{\gamma+1}\right) \tag{19}$$

and the final point

$$\frac{dw}{dv} = \frac{3b^2}{2}\left[(v - v_2)\left(\frac{2\gamma}{\gamma+1} v - v_2\right) - \frac{S_2 A^4}{2\sigma b C_2^8}\right]\frac{v_s - v}{w - \theta^4}. \tag{20}$$

Figure 3 shows a picture of the integral curves of Eq. (17) in the vicinities of the points of the initial and final states. The point (v_1, w_1) for Eq. (19) is a saddle-point singularity. The directions of the separatrices leaving this point are given by the formula [5]

$$\left(\frac{dw}{dv}\right)_1 = a \pm [a^2 + 3b^2 (v_s - v_1)^2]^{\frac{1}{2}},$$

$$a = 2v_1^3 (1 - v_1)^3 (1 - 2v_1).$$

The point (v_2, w_2) is not a singular point of Eq. (20). The radiation energy density w must be a continuous function of x and, therefore, of v also. It is obvious from the adiabats of the final states that at first shock, compression, and therefore, some heating of the gas occur behind the front of the WC. In Fig. 3 this means that at first the state of a particle of air should vary along a separatrix having a negative slope angle in the direction 1'. Then it abruptly assumes some intermediate value with some overcompression $v'_2 (v'_2 < v_2)$ and only after this does it continuously expand along the integral curve up to the final state v_2. Only such a sequence of processes satisfies the solution of Eq. (20), as follows from the form of the integral curves in the phase plane. However, analysis of the adiabat of final states does not admit even this, single possible method of cooling, satisfying the initial formulation of the problem.

Indeed, one can see from Fig. 2 that no intermediate overcompressed states v'_2 belong to the adiabat of final states and therefore they do not exist. The state of a particle of air changes abruptly from the point 0 into the point A. Then, as a result of the subsequent "additional compression" the particle cools to the temperature T_2 at the point B. From here, we can draw the following conclusion. Analysis of the solutions of the complete system of differential equations of gas dynamics and radiation transfer shows that the assumption that the cooling of the hot volume of air by means of a stationary radiation wave of cooling is incorrect. The indicated system of equations does not have a solution that admits such a method of cooling. For this reason, the approach proposed by Zel'dovich and Raizer in [1, 2], based on the fact that the complete system can be replaced by the equations of energy and radiation transfer because of the smallness of the gas-dynamic airflow arising, cannot be used to study the phenomena of radiation cooling.

What we have said, however, does not answer the question of whether or not there exists a nonstationary WC. For this it is necessary to solve the complete system of nonstationary partial differential equations. Because of the great complexity of this system, this can only be done by numerical methods. Therefore this physical phenomenon requires further investigation.

NOTATION

Here D is the velocity of a stationary wave of cooling; ρ_i, P_i, T_i, W_i, V_i (i = 1, 2) are the density, pressure, temperature, specific enthalpy, and specific volume, respectively; u is the velocity of the air flowing out of the front of the WC in the coordinate system fixed in the front; C_1 and C_2 are integrals of the equations of conservation of mass and momentum; S is the radiation flux; σ is the Stefan–Boltzmann constant; R is the universal gas constant; μ_1 is the molar mass of air; c_p and c_V are the heat capacity of air at constant pressure and constant volume; c_1 is the velocity of sound in the hot region; κ_ν is the absorption coefficient of air at the frequency ν; I_ν is the intensity of the emission at the frequency ν; μ is the cosine of the angle between the characteristic and the Ox-axis; κ is the frequency-averaged absorption coefficient of air; I is the frequency-averaged radiation intensity; l is the average mean free path of the radiation; U is the average radiation density; U_{ei} is the equilibrium radiation density; c is the velocity of light in vacuum; v is the dimensionless velocity; w is the dimensionless radiation density; θ is the dimensionless temperature; and, $M_1 = D/c_1$ is the Mach number.

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